IONIZING SHOCK-WAVE PROPAGATION IN A HOMOGENEOUS MAGNETIC FIELD

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Propagation of an ionizing shock wave from a point source in a homogeneous magnetic field is considered. An approximate solution of the equation describing the shock wave surface is obtained. Propagation of an ionizing shock wave within a magnetic field is met in a number of fields within physics: astrophysics [1, 2], laser thermonuclear synthesis [3], space experiments [4, 5], etc. In [6] a self-similar solution was obtained for the front surface of a shock wave propagating from a point source in the atmosphere. The effects of atmospheric inhomogeneity on shock-wave motion were considered in [7, 8]. When the effect of magnetic field is not considered, the problem of ionizing shock-wave propagation in a gas does not differ from those considered in [6-9]. The present study will consider the effect of a homogeneous magnetic field upon propagation of a strong shock wave from a point source.

It is well known [10] that complete ionization of air occurs at Mach numbers equal to 14-20, i.e., at $p_2/p_1 \approx 200$-500 (where p_1 is the pressure of the neutral gas ahead of the shock wave front and p_2 is the gas pressure behind the front). At a height of $\sim 3 \cdot 10^5$ m the unperturbed air pressure $p_1 \sim 10^{-5}$ N/m^2, so that ionization in the air occurs at $p_2 \sim 10^{-3}$ N/m^2. The maximum "magnetic" pressure on the ionized gas due to the earth's magnetic field $H_0 = 0.5$ Oe is $\sim 10^{-3}$ N/m^2. From these values it follows that at an altitude of $\sim 3 \cdot 10^5$ m the effect of magnetic field upon ionizing shock wave propagation must be considered. Since the free path length of the charged particles is ~ 10 m and the ionic Larmor radius is ~ 10 m, the hydrodynamics approximation for a shock wave front radius $\sim 10^3$ m or higher is well satisfied. The height of a standard atmosphere at $\sim 3 \cdot 10^5$ m is equal to $\sim 1.5 \cdot 10^4$ m [10], so that for a shock-wave front radius of $\sim 10^3$ m the effect of atmospheric inhomogeneity can be neglected.

Magnetic field perturbation by a strong spherical shock wave was considered in [11]. In [12] it was shown that the magnetic field penetrates beyond the shock-wave front relatively slowly. For a plasma of infinitely high conductivity, the magnetic field behind the front is negligibly small [13, 14].

Considering the above, we write the Rankin—Hugoniot conditions relating the gas parameters on the two sides of the discontinuity in the form [15, 16]

$$D^2 = \rho_2\rho_1^{-1}(p_2 - p_1 - H^2/8\pi)(\rho_2 - \rho_1)^{-1}; \tag{1}$$

$$\varepsilon_2 - \varepsilon_1 + 0.5(\rho_1 - \rho_2)\rho_2^{-1}\rho_1^{-1}(p_2 + p_1 + H^2/8\pi) = 0, \tag{2}$$

where D is the shock-wave velocity, p_1 and p_2 are the hydrodynamic pressures, ρ_1 and ρ_2 are the densities, and ε_1 and ε_2 are the internal energies of the gas ahead of and behind the shock-wave front. Energy dissipated in gas ionization is neglected in Eqs. (1), (2).

Using the equation of state of an ideal gas for the plasma and neglecting hydrodynamic pressure ahead of and behind the shock-wave front, we find the relationship between p_2 and p_1:

$$\rho_2 = \rho_1[1 + 2(\gamma - 1)^{-1}(1 + H^2/(8\pi p_2))^{-1}]. \tag{3}$$

We write the equation for shock-wave velocity in the magnetic field in the form

$$D^2 = 0{,}5p_2(\gamma - 1)\rho_1^{-1}[1 - H^4/(8\pi p_2)^2][1 + 2(\gamma - 1)^{-1}(1 + H^2/(8\pi p_2))^{-1}]. \tag{4}$$

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In the special case H = 0 we obtain from Eq. (4) the velocity of a strong shock wave in a gas [5, 10] $D^2 = 0.5p_2(1+\gamma)\rho_1^{-1}$.

In the case where the ratio of the "magnetic" pressure of the field to the hydrodynamic pressure on the front is small ($\beta \gg 1$, $\beta = 8\pi p_2 H^{-2}$), Eq. (4) simplifies:

$$D^2 \simeq 0.5p_2(1+\gamma)\rho_1^{-1} - H^2(8\pi\rho_1)^{-1}. \tag{5}$$

Following [7, 17], we will assume that the entire mass of ionized gas moves together with the shock-wave front. We will denote the equation of the front surface by $f(r, \theta, t) = 0$. Then the shock-wave velocity can be written as

$$D = -\frac{\partial f}{\partial t}|\nabla f|^{-1}. \tag{6}$$

For the gas pressure p_2 we use the expression [7]

$$p_2 = \lambda E_0(\gamma - 1)/V(t), \tag{7}$$

where E_0 is the total energy liberated at the point source, $V(t)$ is the volume of ionized gas occupied by the wave, and $\lambda = \lambda(\gamma)$ is a coefficient which characterizes the ratio of the energy density in the vicinity of the shock-wave front to the mean energy density over volume [7]. Here, as in [7], it is assumed that λ is constant over the entire surface.

We will consider the case of low "magnetic" pressure, so that with consideration of Eq. (5), Eq. (6) for the shock-wave surface can be written in the form

$$\left(\frac{\partial f}{\partial t}\right)^2 = (\nabla f)^2\left[0.5p_2(1+\gamma)\rho_1^{-1} - H^2(8\pi\rho_1)^{-1}\right]. \tag{8}$$

Moreover, we assume [8] that the equation of the shock-wave front is soluble in a spherical coordinate system relative to the front radius, $r = r(\theta, t)$. With consideration of Eq. (7), Eq. (8) takes on the form

$$2m\left(\frac{\partial r}{\partial t}\right)^2 = (1-\alpha h^2)\left[1+\frac{1}{r^2}\left(\frac{\partial r}{\partial \theta}\right)^2\right], \tag{9}$$

where $m(t) = \rho_1 V(t)(\gamma^2-1)^{-1}(\lambda E_0)^{-1}$; $h^2 = H^2 H_0^{-2}$; $\alpha(t) = H_0^2 m(4\pi\rho_1)^{-1}$; H_0 is the magnetic field intensity at infinity. For $\beta \gg 1$ we have $\alpha \ll 1$.

The initial condition for Eq. (9) is the equation of the surface bounding the wave volume at the initial time.

The magnetic field outside the volume through which the shock wave has traveled is determined from the magnetostatics equations with boundary conditions on the moving surface separating the ionized gas and magnetic field [11, 12] and constant field at infinity. Since the ratio of magnetic to hydrodynamic pressure is low ($\alpha \ll 1$), we will seek a solution of Eq. (9) in the form of an expansion in powers of the small parameter

$$r(\theta, t) = r_0(t) + \sum_{k=1}^{\infty} \alpha^k r_k(\theta, t).$$

Equations for calculation of $r_0(t)$, $r_1(\theta, t)$, etc. can be obtained from Eq. (9),

$$\partial r_0/\partial t = (2m)^{-1/2}; \tag{10}$$

$$\partial r_1/\partial t = -0{,}5h^2(2m)^{-1/2}. \tag{11}$$

The zeroth approximation at $\alpha h^2 = 0$ is independent of angle

$$r_0(t) = At^{2/5}, \quad A = 0{,}93\left(E_0\rho_1^{-1}\right)^{1/5}$$

and coincides with the solution of the self-similar problem of shock-wave propagation in the atmosphere presented in [6].

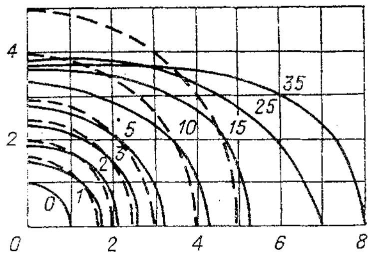

Fig. 1

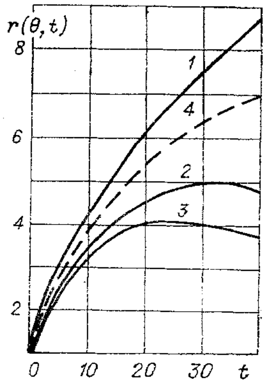

Fig. 2

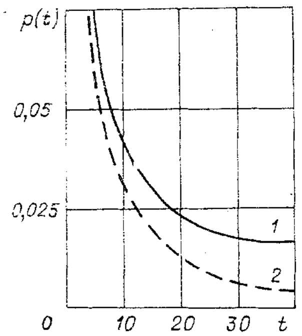

Fig. 3

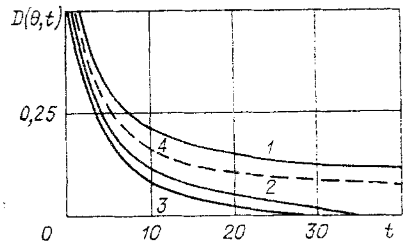

Fig. 4

Substituting the solutions of the magnetostatics equations [15, 18] $h = (1 + 0.5r_0^3 r^{-3}) \sin\theta$ at $r = r_0(t)$ in Eq. (11), we find

$$r_1(\theta,\ t) = -(9/8)At^{2/5}\sin^2\theta.$$

To the accuracy of a second approximation, the solution of Eq. (9) for the surface of an ionizing shock wave propagating in a homogeneous axisymmetric magnetic field has the form

$$r(\theta,\ t) = At^{2/5}\left[1 - (3/8)H_0^2A^3t^{6/5}(\lambda E_0)^{-1}(\gamma^2-1)^{-1}\sin^2\theta\right]. \tag{12}$$

It is evident from Eq. (12) that at a fixed moment of time the shock-wave front in the second approximation is an ellipsoid.

In the case where the ratio of "magnetic" to hydrodynamic pressure is not small, Eq. (6) was solved numerically by the finite-difference method in a spherical coordinate system with consideration of Eq. (4).

For this purpose the differential operators in Eq. (6) were replaced by differences. The derivative with respect to angle $r^{-1}\partial r/\partial\theta$ was replaced by centrally symmetric differences, so that the accuracy of the calculations with respect to coordinate is of the order of $O(h_\theta^2)$ (where h_θ is the step in angle), while the time accuracy is $O(\tau)$ (where τ is the step in time).

The problem was numerically solved with the following parameter values: the shock-wave front radius at the initial time was taken as $r(t = 0) = 1$, the "magnetic"/hydrodynamic pressure ratio at the initial time $\beta = H_0^2V(\gamma - 1)(8\pi\lambda E_0)^{-1} = 10^{-1}$, the initial energy density $C = (\gamma-1)(\lambda E_0)^{1/2}(2\rho_1 V)^{-1/2} = 1$. We note that these arbitrary parameter values do not limit the generality of the solution.

Calculations were performed with an angle θ varying from 0 to $\pi/2$ in steps $h_\theta = \pi/360$. The precision of the calculations was maintained by variation of the size of the steps in time and angle. Under the influence of the magnetic field the shock-wave front deforms, so that the calculations considered the change in the angle of the normal to the surface as compared to the spherically symmetric case by an amount $\varphi = \arctan(r^{-1}\partial r/\partial\theta)$.

Figures 1-4 show some results of numerical solution of the problem of shock-wave propagation from a point source in a homogeneous magnetic field.

Figure 1 shows curves of $r(\theta, t)$ for various moments in time. The dashed curves correspond to ionizing shock-wave propagation at times 1, 2, 3, 5, 10, 15 without magnetic field. The solid curves describe the shock-wave front at $t = 0, 1, 2, 3, 5, 10, 15, 25, 35$ sec with consideration of magnetic field. It is evident that at $t \simeq 25$ the front motion in the direction perpendicular ($\theta = \pi/2$) to the unperturbed magnetic field intensity is halted.

Curves of $r(\theta, t)$ as a function of time are shown in Fig. 2 for $\theta = 0$ (curve 1), $\theta = \pi/4$ (curve 2), $\theta = \pi/2$ (curve 3). For comparison, Fig. 2 also shows $r(t)$ (curve 4) for propagation of a spherically symmetric shock wave [6]. In the direction $\theta = 0$, for a fixed t, the shock-wave transverses a greater distance than in the spherically symmetric case. This is because, due to deformation of the front surface by ponderomotive forces, the volume of ionized gas at a given time is always smaller than in the spherically symmetric case, so that the gas pressure on the front is higher.

Figure 3 shows gas pressure on the front as a function of time with (curve 1) and without (curve 2) consideration of magnetic field.

Figure 4 shows shock-wave front velocity as a function of time for various directions: $\theta = 0$ (curve 1), $\theta = \pi/4$ (curve 2), $\theta = \pi/2$ (curve 3). Curve 4 corresponds to the spherically symmetric case. At certain times ($t = 25$ at ($t = 25$ at $\theta = \pi/2$, $t = 35$ at $\theta = \pi/4$, etc.) the front velocity becomes equal to zero. Assuming the validity of conditions on the discontinuity, the numerical calculations were performed for the case where the "magnetic" pressure exceeds the ionized gas pressure on the front. Due to the large ponderomotive forces, even at $p_2 \gg p_1$, the front velocity in the direction perpendicular to the magnetic field may be negative. In later stages of wave propagation (at $p_2 \sim p_1$), apparently pulsations of the ionized gas "disk" occur.

It is evident from Fig. 4 that the ionizing shock-wave front velocity at $\theta = 0$ is higher than in the spherically symmetric case.

It is not possible to determine the time when the front halts in the direction $\theta = \pi/2$ from solution of Eq. (12). However, from dimensional considerations it follows that the corresponding time is given by $t_0 = kE_0^{1/3}\rho_1^{1/3}H_0^{-5/3}$. Numerical calculations give $k \simeq 40$.

In conclusion, we will note the correspondence between the model considered and the actual physical process. In reality the gas pressure on the shock-wave front is angle-dependent, which leads to a reduction of the magnetic field's effect on wave characteristics.

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A NUMERICAL INVESTIGATION OF THE ELECTRICAL CHARACTERISTICS OF THE ELECTRODE BOUNDARY LAYER OF A SLIGHTLY IONIZED PLASMA OF MOLECULAR GASES

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The hydrodynamic problem of determining the electrical characteristics of the electrode region in a slightly ionized plasma in chemical equilibrium was formulated in [1] and subsequently analyzed more than once. The present article is devoted to a numerical solution of this problem. We note that, besides the independent interest, such a solution is also of interest for estimating the degree of accuracy of various approximate approaches.

The problem under consideration is a boundary-value problem for a system of nonlinear, ordinary differential equations; for the conditions of practical interest this system contains two small parameters to the leading derivatives, while in the case of a relatively low electrode temperature it also contains a third small parameter in the exponent. Certain difficulties arise in the direct numerical solution of problems of this type, and therefore one or another simplifying assumptions were made in [2-4], devoted to the numerical solution of this problem. For example, in [2, 3] the electrode layer is subdivided into a space-charge layer and a quasineutral region, and the solution of the problem is sought separately in each region with subsequent joining. In this case the ionization of neutral atoms and the recombination of charged particles in the space-charge layer are ignored, which prevents a correct description of the behavior of the volt—ampere characteristic curves of the electrode region of molecular-gas plasma for high densities of current to the electrode [5]. Some important terms of the system of determining equations were not taken into account in [4], and in [2-4] the problem was solved by the shooting method.

An efficient iteration algorithm based on the trial-and-error method is developed in the present article to obtain a direct numerical solution of the problem under consideration. Calculation results are given for the case of a plasma of combustion products with a potassium admixture and a wide range of electrode temperatures, and a detailed comparison is made with the results of calculations by the analytical equations of [5], obtained by the method of joined asymptotic expansions, and with experimental data.

1. Statement of the Problem

Let us consider a multicomponent, slightly ionized plasma of molecular gases containing neutral components, positive singly charged ions of atoms of one of the neutral components

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